

Profit Predicting

Task

Introduction of the Problem/Context: In the context of randomly guessing answers to questions on a true-false quiz, students will find experimental and theoretical probabilities in 50-50 probability situations.

Selected GPS:

- **M6D2.** Students will use experimental and simple theoretical probability and will understand the nature of sampling. They will also make predictions from investigations.
 - a. Predict the probability of a given event through trials/simulations (experimental probability), and represent the probability as a ratio.
 - b. Determine, and use a ratio to represent, the theoretical probability of a given event.
- **M8D2.** Students will determine the number of outcomes related to a given event.
 - a. Use tree diagrams to find the number of outcomes.
- **M8D3.** Student will use the basic laws of probability.
 - a. Find the probability of simple independent events.
 - b. Find the probability of compound independent events.
- **M6P3.** Students will communicate mathematically.
 - a. Organize and consolidate their mathematical thinking through communication.
 - b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
 - c. Analyze and evaluate the mathematical thinking and strategies of others.
 - d. Use the language of mathematics to express mathematical ideas precisely.
- **M6P5.** Students will represent mathematics in multiple ways.
 - a. Create and use representations to organize, record, and communicate mathematical ideas.
 - b. Select, apply, and translate among mathematical representations to solve problems.
 - c. Use representations to model and interpret physical, social, and mathematical phenomena.

Classroom Materials: Lesson Outlines

Profit Predicting

Grade Level/Subject: 6th Grade (can be applied to grades 7-8), Experimental and Theoretical Probabilities in Binomial Probability Situations

Overview: In the context of randomly guessing answers to questions on a true-false quiz, students will find experimental and theoretical probabilities in 50-50 probability situations.

Key Objectives:

- To use tree diagrams to find the number of outcomes (GPS: M8D2a).
- Predicting probability through simulation (GPS: M6D2a).
- Using ratios to express theoretical and experimental probabilities (GPS: M6D2b).





Possible Materials:

- Calculators
- A transparency or handouts of the quiz and the quiz results.

Task: Tell the students that they are to take a four question, true/false pop quiz written in a secret code. Let the class see the quiz, and ask them questions about how they think they will do on the quiz. For example: How many do they expect to get right? Who do they think will do the best? Is each student equally likely to do as well as the others? Is there a strategy that would help them increase their score, or would random guessing do as well as any strategy?

Pop Quiz!

Tell whether each statement is true or false:

1. 
2. 
3. 
4. 

After having them think about this scenario for a while, have them take the quiz, then give them the answers and have them grade themselves (the answers are TFFT). Have the students remember their scores on the quizzes, as they will be used at the end of the lesson.

Sample Questions:

The following data is from two other classes that took the quiz. This data is included on the website as a black line master. The correct answers for the quiz are TFFT.

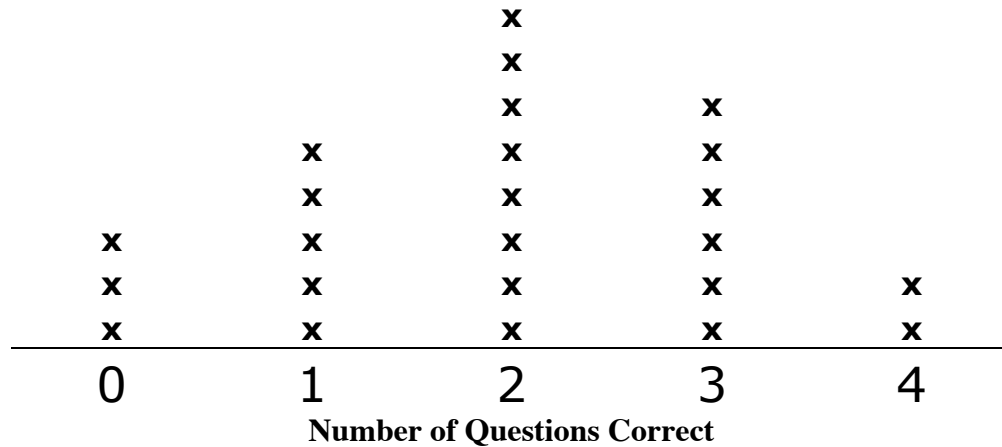
FFFF	TFFT	TTTT	FTFF	FFFT	TTTT
TFFT	TFTF	FFTF	FFTT	FFFF	TTTF
TFTT	FTTF	TFFF	TFFT	FFFT	TTFT
FTTT	TTTF	FTFT	TFFF	FTFT	TTFF
TFTF	FFTT	TTFF	TFFF	FETF	TFTT
TTTT	TFTF	FFFT	FTTF	FTTT	TFFF
FFTF	FFFT	TFFF	FTTT	FTTF	FFFF
FTFF	TFFF	FTFT	TFTF	FFTT	TFFT
FFTF	FFTT	TTTF	TFTT	TTFE	FFTT
TFTF	FFTF	FTFF	TTTT	TFTF	TTFT

- Next to each set of four answers, write down the number of problems each student got correct.
- By using your work from problem 1, calculate the experimental probability that a student who guesses every answer will get all four answers right.
- What is the experimental probability that someone who guesses every answer will get exactly three answers right?
- Find the experimental probability that someone who guesses every answer will get exactly two answers right.
- Find the experimental probability that someone who guesses every answer will get exactly one answer right.
- Find the experimental probability that someone who guesses every answer will get no answers right.
- Make a tree diagram to find all the possible results on the quiz. For example, RWRR would mean that the student got the second problem wrong and all others right.
- How many different right/wrong combinations are there for this quiz?
- If you guess on every question, are you more likely to get exactly 2 answers correct or exactly 3 answers correct?
- If you guess on every question, are you more likely to get the first answer wrong and the last three right or to get the first two answers wrong and the last two answers right?

11. Is it true that the probability of getting the first answer wrong and the last three right is the same as getting any three particular answers right?

Assessment Ideas:

As a class make a line plot of the number of correct answers from the class data, similar to the one below:



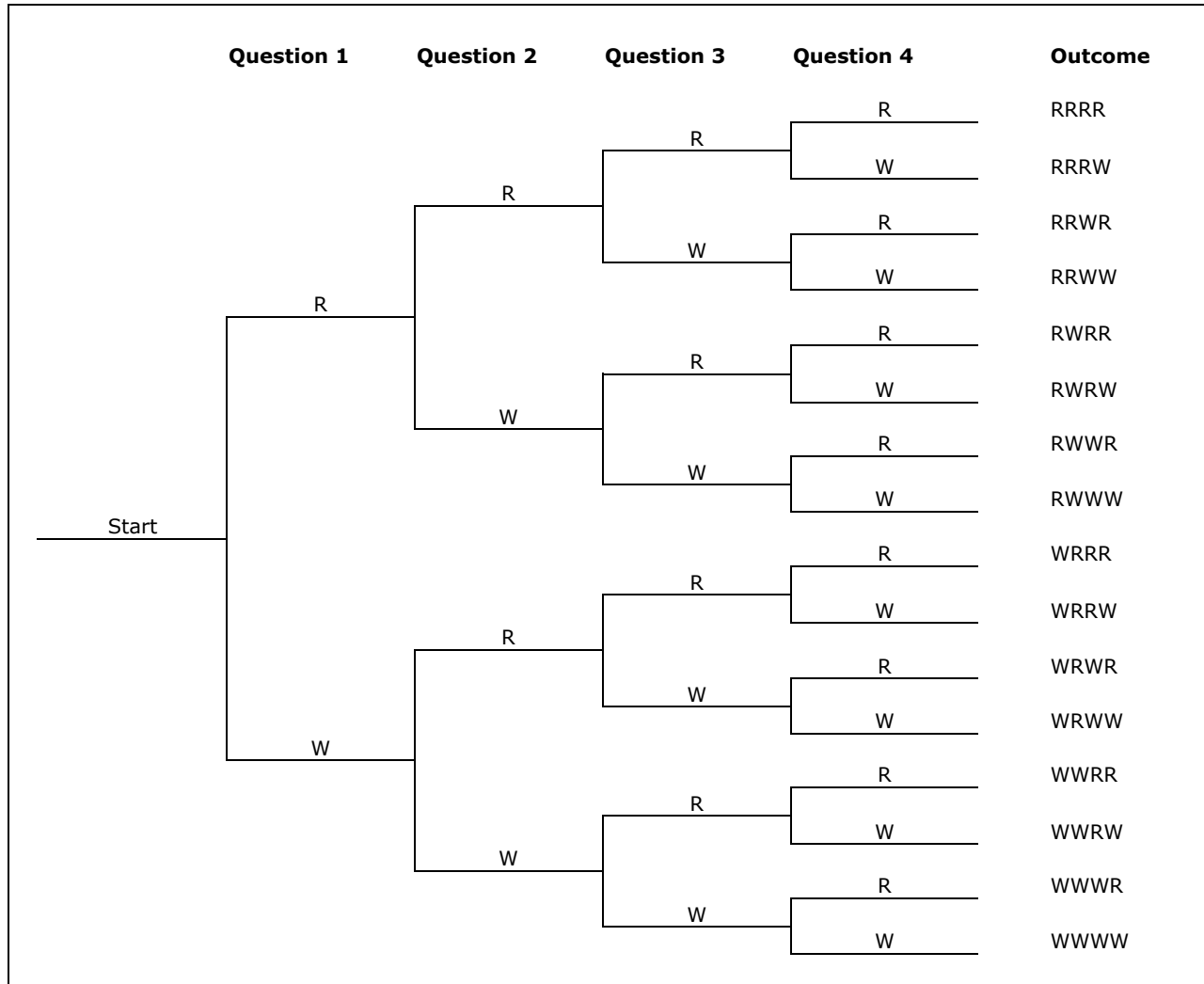
- Have students find the experimental probabilities of getting 0, 1, 2, 3, and 4 problems correct on the quiz using the class data.
- Have students compare the analysis of the 60 quizzes in problems 1-6 to the analysis of the class data. Which set of data would they expect to yield experimental probabilities closer to theoretical probabilities? The sample from the class, or the larger sample of 60 students from problems 1-6?

Sample Question Solutions:

1.

FFFF -1	TFFT -3	TTTT -3	FTFF -0	FFFT -2	TTTT -3
TFFT -3	TFTF -3	FFTF -2	FFTT -3	FFFF -1	TTTF -2
TFTT -4	FTTF -1	TFFF -2	TFFT -3	FFFT -2	TTFT -2
FTTT -2	TTTF -2	FTFT -1	TFFF -2	FTFT -1	TTFF -1
TFTF -3	FFTT -3	TTFF -1	TFFF -2	FTTF -1	TFTT -4
TTTF -2	TFTF -3	FFFT -2	FTTF -1	FTTT -2	TFFF -2
FFTF -2	FFFT -2	TFFF -2	FTTT -2	FTTF -1	FFFF -1
FTFF -0	TFFF -2	FTFT -1	TFTF -3	FFTT -3	TEFT -3
FFTF -2	FFTT -3	TTTF -2	TFTT -4	TTFE -1	FFTT -3
TFTF -3	FFTF -2	FTFF -0	TTTT -3	TFTF -3	TTFT -2

2. $2/60 = 1/30$
3. $18/60 = 3/10$
4. $23/60$
5. $13/60$
6. $3/60 = 1/20$
- 7.



8. 16
9. There are 6 ways of getting exactly 2 answers correct, so the probability of getting exactly 2 answers is $6/16$. There are 4 ways of getting exactly 3 answers correct, so the probability of getting exactly 3 answers correct is $4/16$. You are thus more likely to get exactly 2 answers correct when guessing.
10. Each of these quiz results could only happen in one way, so they both have a $1/16$ possibility of happening, which means they are equally likely to occur.
11. No, because there are 4 ways of getting any three particular answers right, but only one way to get the first answer wrong and the last three right.

* Lesson adapted from Connected Mathematics: What Do You Expect?: Guessing Answers, © 2002 by Prentice Hall.