The Orange Juice Problem:

Subject: Ratios and Proportional Reasoning

Grade Level: 6th Grade (can be applied to grades 7-8).

Overview:
This investigation engages students in using proportional reasoning to determine a) which of the four orange juice mixtures contains the highest and the lowest percent of orange concentrate and b) the orange juice recipe for each of the four mixtures if 120 cups are needed.

GPS Addressed:
M6N1. Students will understand the meaning of the four arithmetic operations as related to positive rational numbers and will use these concepts to solve problems.

- c. Determine the greatest common factor (GCF) and the least common multiple (LCM) for a set of numbers.
- f. f. Use fractions, decimals, and percents interchangeably.
- g. Solve problems involving fractions, decimals, and percents.

M6A1. Students will understand the concept of ratio and use it to represent quantitative relationships.
M6A2. Students will consider relationships between varying quantities.

- b. Use manipulatives or draw pictures to solve problems involving proportional relationships.
- c. Use proportions (a/b=c/d) to describe relationships and solve problems, including percent problems.
- g. Use proportional reasoning (a/b=c/d and y=kx) to solve problems.

M6P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

M6P2 Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

M6P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

M6P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.

b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

M6P5. Students will make connections among mathematical ideas and to other disciplines.
a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.

Classroom Materials:
Orange Juice Concentrate
Water
Large Mixing Containers
Small measuring cups to hold water
Small 2-3 ounce cups (bathroom cup)
Spoons to stir
Orange and white stock cards (optional)

Key Learning Outcomes:

- To compare different ratios, percentages, or fractions.
- To recognize part-to-whole and part-to-part relationships.
- To understand and use scaling factors.
- To solve problems by gathering data, using multiple strategies, making and justifying conjectures.
- To recognize multiple ways of representing ratio or fractions.

Task:
Launch: Have you ever made orange juice from frozen orange concentrate? Describe what you had to do. Today, we are in charge of making orange juice for a group of campers, but we first have to decide which orange juice recipe to use.

Investigation: Students are divided into four groups for the "mixing juice" portion of the lesson. Each group is given one of the following recipes along with the materials needed to make orange juice:

Mix A
2 cups concentrate
3 cups cold water

Mix B
1 cup concentrate
4 cups cold water

Mix C
4 cups concentrate
8 cups cold water

Mix D
3 cups concentrate
5 cups cold water

The first task is to decide within their group on the process of how to make their orange juice recipe. Once each group has completed the task, the groups will rotate around the room tasting each of the different mixtures, making notes and discussing the differences they notice.

Next, the students will prove mathematically which juice is more orangey? Which one is least orangey? Have them explain their answers.

Lastly, students will determine **for each recipe** how much concentrate and how much water are needed to make juice for 240 campers. Each camper will get ½ cup of juice. Have the students explain their answers.

It is helpful to see a variety of strategies for solving this problem. Once students have worked through and explained their solutions, other examples of student work can be shown. Using the samples of student work (oj_transparency.pdf), have the students discuss how these strategies worked. Students can reflect on their process in comparison to these other methods, which would they choose to use in the future? What benefits do they gain from using a different method? The key point here is that there are a number of different ways to approach this problem.

Other Sample Questions:

1) Does the order of ingredients matter in determining the orange concentration of a mixture?

2) How important is it to obtain an accurate measurement in a recipe?
What are some ways to be sure that the right amount is measured each consistently?

3) What kind of relationship (part to part or part to whole) would the following examples represent?

a) In a taste test people who prefer older cola out-numbered those who preferred Colo-nola by a ratio of 3 to 2.
b) The ratio of boys to girls in our class is 12 boys to 15 girls
c) The ratio of boys to students in our class is 12 boys to 27 students.
d) A paint mixture calls for 5 parts blue paint and 2 parts yellow paint.
e) The ratio of kittens to cats in our neighborhood is 1 to 4.

4) Does the mixture with the most orange concentrate is necessarily the one that tastes the most orangey? What if two mixtures have the same amount of orange concentrate? Will they taste the same?

5) What fraction of each mixture is orange concentrate?

6) Is it possible find out which mixture is most or least orangey by comparing the difference between the numerator and denominator from the fraction in question 5? (e.g. Mixture A has an orange concentrate fraction of 2/5, Mixture D has an orange concentrate fraction of 3/8. The

difference of numerator and denominator of mixture A is 3, whereas mixture D is 5. Thus, mixture A is more orangey then mixture D).

7) What are some different ways that we can model the orange concentration of each mixture?

Solutions to Sample Questions:

1) No.
2) Yes. Possible answers: use a tape on the cup, fill it to the rim, etc.
3) a. part to part b. part to part c. part to whole d. part to part
   e. depends how kittens are defined.
4) No. No. Depends on how much water is in the mixture.
5) Mix A: 2/5, mix B: 1/5, mix C: 1/3, and mix D: 3/8.
6) It will not always work. Counterexample: ½ and 3/5.
7) Possible answers: draw pictures, use manipulatives, etc.